正压大气非线性波的二级近似(一)

——涡度方程和散度方程的求解

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提要

本文是用摄动法求取正压原始方程一类非线性大尺度慢波解的二级近似的第一部分，通过求解二级问题中的涡度方程和散度方程，给出了速度势和非地转位势的二级修正的解析表达式，并计算了准地转性参数 $\varepsilon = 0.3$ 时散度和非地转涡度的分布.

一、引言·二级问题

摄动法是研究非线性问题的一种有效方法，合理地运用这种方法不仅能给出足够正确的解的解析结构，用来进行定性讨论，甚至还能获得相当精确的近似解析解，用于定量的研究.

一般，摄动法要求将无量纲函数 F 按无量纲小参数 ε 展开

$$F \sim F_0 + \varepsilon F_1 + \varepsilon^2 F_2 + \cdots \tag{1}$$

我们称 (1) 式中关于 ε 的幂级数为摄动级数；称 F_0 为 F 的零级近似，F_1 为一级修正，$F_0 + \varepsilon F_1$ 为一级近似，…等；而将 F_0、F_1……所满足的问题分别称为零级问题、一级问题……等.对于一个非线性问题来说，其零级问题可能还是非线性的，但比原问题简单易解；在求得零级近似后，自然会按照由低到高的顺序去求解一级以上的问题，此时它们都表现为线性问题，原则上总可解出. 但是，随着问题的"级"别的提高，求解过程的工作量将急剧增加，很快就达到手工运算所无法胜任的地步. 这是因为在一级以上的问题中，原来的非线性项不仅表现为零级近似与本级修正之间的作用，而且还表现为较低级的修正之间的作用，只不过它们分别以变系数线性项和强迫项的形式出现罢了.

以下我们围绕用摄动法求正压原始方程组非频散的大尺度慢波解的问题，援引[1]中的结果来说明此点，并为求解二级问题作好准备.

对"深水"条件下的准地转大尺度低速流，适当选取特征量，使得准地转性参数 $\varepsilon(\varepsilon < 1)$ 和 Rossby 数相等，将正压原始方程组无量纲化(如无特别说明，以下出现的量均为无量纲量). 考虑在 β 平面上的纬向通道 $0 \leqslant y \leqslant \pi$ 内以常相速 C 沿 x 方向平移且

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在南北边界上满足刚壁条件的非频散波解，则流函数 ψ、速度势 χ 和非地转位势 A 皆可表为 $\xi = x - ct$ 和 $\eta = y$ 的二元函数，将它们和相速 C 按 (1) 对 ε 展开，由摄动法容易得出

$$\chi_0 = A_0 = 0 \tag{2}$$

而零级问题归结为准地转位涡度方程的特征值问题

$$\begin{cases} J(c_0\eta + \psi_0,\ (\Delta - \mu^{-2})\psi_0 + \beta\eta) = 0 & (3) \\ \left.\dfrac{\partial\psi_0}{\partial\xi}\right|_{\eta=0,\pi} = 0 & (4) \end{cases}$$

迭加在平直西风上的 Rossby 波

$$\psi_0 = -U\eta + {}_0\psi \sin m\xi \cdot \sin n\eta \tag{5}$$

$$c_0 = U + \frac{\beta + \mu^{-2}U}{\kappa_{11} - \mu^{-2}} \tag{6}$$

是它的一类解，其中 n 为整数，$\kappa_{11} = -(m^2 + n^2)$. 我们就取(2)、(5)、(6)式为正压原始方程非线性波解的零级近似。

一级问题由以下三个问题组成

$$\begin{cases} \Delta\chi_1 = c_0\dfrac{\partial}{\partial\xi}\Delta\psi_0 - J(\psi_0,\ \beta\eta + \Delta\psi_0) & (7) \\ \left.\dfrac{\partial\chi_1}{\partial\eta}\right|_{\eta=0,\pi} = 0 & (8) \end{cases}$$

$$\begin{cases} \Delta A_1 = (\beta\eta + \Delta\psi_0)\Delta\psi_0 + \nabla\psi_0\cdot\nabla(\beta\eta + \Delta\psi_0) - \Delta\left(\dfrac{u_0^2 + v_0^2}{2}\right) & (9) \\ \left.\dfrac{\partial A_1}{\partial\eta}\right|_{\eta=0,\pi} = \left[-\dfrac{\partial\chi_1}{\partial\xi} - \beta\eta u_0\right]_{\eta=0,\pi} & (10) \end{cases}$$

$$\begin{cases} \mathscr{L}(\psi_1) = -\dfrac{\partial}{\partial\xi}\left[c_1(\Delta - \mu^{-2})\psi_0 - c_0\mu^{-2}A_1\right] + (\beta\eta + \Delta\psi_0 - \mu^{-2}\psi_0)\Delta\chi_1 \\ \qquad + J(\psi_0, -\mu^{-2}A_1) + \nabla\chi_1\cdot\nabla(\beta\eta + (\Delta - \mu^{-2})\psi_0) & (11) \\ \left.\dfrac{\partial\psi_1}{\partial\xi}\right|_{\eta=0,\pi} = 0 & (12) \end{cases}$$

其中 $u_0 = -\dfrac{\partial\psi_0}{\partial\eta}$, $v_0 = \dfrac{\partial\psi_0}{\partial\xi}$. 而

$$\mathscr{L}(F) = c_0\frac{\partial}{\partial\xi}(\Delta - \mu^{-2})F - J(\psi_0,\ \Delta F) - J(F,\ \beta\eta + \Delta\psi_0) \tag{13}$$

(7)、(9)和(11)式分别是一级修正所满足的涡度方程、散度方程和位涡度方程，它们都是线性方程，但项数已比零级问题显著增加，尤其是 (11) 式，它的右端不仅由于非线性作用增加了项数，而且还依赖于本级问题中前二问题的解，因而计算量更大。

为书写简便计，引进记号

$$\begin{cases} (c_ic_j) = \cos im\xi\cdot\cos jn\eta \quad (c_is_j) = \cos im\xi\cdot\sin n\eta \\ (s_ic_j) = \sin im\xi\cdot\cos jn\eta \quad (s_is_j) = \sin im\xi\cdot\sin jn\eta \end{cases} \tag{14}$$

可求得一组一级修正

$$\chi_1 = c_0\mu^{-2}/\kappa_{11}m\cdot{}_0\psi(c_1s_1) + {}_1X_1^+(c_1c_0)e^{m\eta} + {}_1X_1^-(c_1c_0)e^{-m\eta} \tag{15}$$

$$A_1 = -\frac{1}{2}\beta U\eta^2 - \beta/\kappa_{11}n\cdot {}_0\phi(s_1c_1) + \frac{1}{4}n^2\cdot {}_0\phi^2(c_2c_0) + \frac{1}{4}m^2\cdot {}_0\phi^2(c_0c_2)$$
$$+\beta\cdot {}_0\phi(s_1s_1)\eta + {}_1X_1^+(s_1c_0)e^{m\eta} - {}_1X_1^-(s_1c_0)e^{-m\eta} \tag{16}$$
$$\phi_1 = -{}_1U_1\eta - {}_1U_2\eta^2 + ({}_1X_1^+ - {}_1X_1^-)(s_1c_1) + {}_1\Psi_{20}^{cc}(c_2c_0) + {}_1\Psi_{02}^{cc}(c_0c_2)$$
$$+{}_1B_0(s_1c_1)\eta + {}_1B_1(s_1s_1)\eta + {}_1B_2(s_1c_1)\eta^2 + {}_1d_1(c_2c_0)e^{l\eta}$$
$$+{}_1d_2(c_2c_0)e^{-l\eta} + {}_1X_1^+(s_1c_0)e^{m\eta} + {}_1X_1^-(s_1c_0)e^{-m\eta} \tag{17}$$
$$c_1 = \frac{1}{\kappa_{11}-\mu^{-2}}\left[\kappa_{11}\cdot {}_1U_1 - \kappa\mu^{-2}(c_0-U)\left(\kappa_{11}U+\frac{1}{2}\beta\right)\right] \tag{18}$$

其中 $l=\sqrt{\kappa_{11}+4m^2}$, ${}_1X_1^+$、${}_1X_1^-$、${}_1\Psi_{20}^{cc}$、${}_1\Psi_{02}^{cc}$、${}_1U_2$、${}_1B_0$、${}_1B_2$、${}_1d_1$、${}_1d_2$ 等是与 β、μ^{-2}、m、n、${}_0\phi$、U 及 c_0 有关的常数，${}_1U_1$ 是一个自由参数。

我们的研究[2]表明，对于 $\varepsilon=0.1$ 的情形，一级近似比零级近似有明显的改进. 以下我们来求取二级修正，这不仅是了解摄动级数的性质所需要的，而且对于描写某些初值条件所引起的波动也是必要的.

二级问题仍由涡度方程、散度方程和位涡度方程三部分及相应的边界条件组成

$$\begin{cases}\Delta\chi_2 = \mu^{-2}\left[c_0\dfrac{\partial\phi_1}{\partial\xi} + c_0\dfrac{\partial A_1}{\partial\xi} + c_1\dfrac{\partial\phi_0}{\partial\xi} + J(A_1,\phi_0) - \nabla\chi_1\cdot\nabla\phi_0 - \phi_0\Delta\chi_1\right] \\ \qquad = H_2 & (19)\\ \left.\dfrac{\partial\chi_2}{\partial\eta}\right|_{\eta=0,\pi} = 0 & (20)\end{cases}$$

$$\begin{cases}\Delta A_2 = c_0\dfrac{\partial}{\partial\xi}\Delta\chi_1 + (\beta\eta+\Delta\phi_0)\Delta\phi_1 + \Delta\phi_1\Delta\phi_0 + \nabla\phi_1\cdot\nabla(\beta\eta+\Delta\phi_0) \\ \qquad +\nabla\phi_0\cdot\nabla(\Delta\phi_1) - J(\chi_1,\ \beta\eta+\Delta\phi_0) - \Delta[\nabla\phi_0\cdot\nabla\phi_1 \\ \qquad + J(\phi_0,\ \chi_1)] = Q_2 & (21)\\ \left.\dfrac{\partial A_2}{\partial\eta}\right|_{\eta=0,\pi} = -\left[\dfrac{\partial\chi_2}{\partial\xi} + \beta\eta\left(\dfrac{\partial\chi_1}{\partial\xi} - \dfrac{\partial\phi_1}{\partial\eta}\right)\right]_{\eta=0,\pi} & (22)\end{cases}$$

（位涡度方程暂略）

可以看出，同一级问题相比，二级问题中右端的项数大大增加了，这是求解二级修正的主要困难所在.

二、涡度方程的求解

由已得到的零级近似和一级修正计算(19)式的右端，得到

$$H_2 = \mu^{-2}\Big\{c_1m\cdot {}_0\phi(c_1s_1)$$
$$+\Big[c_0m({}_1X_1^+ - {}_1X_1^- - \beta n/\kappa_{11}\cdot {}_0\phi) + \frac{1}{4}mn(m^2-n^2){}_0\phi^3$$
$$+U(c_0-U)mn\cdot {}_0\phi\Big](c_1c_1) + [c_0m(\beta\,{}_0\phi + {}_1B_1) + Uc_0\mu^{-2}m\cdot {}_0\phi](c_1s_1)\eta$$
$$+c_0m\cdot {}_1B_0(c_1c_1)\eta + c_0m\cdot {}_1B_2(c_1c_1)\eta^2$$
$$+\Big[-2c_0m\left(\frac{1}{4}n^2{}_0\phi^2 + {}_1\Psi_{20}^{cc}\right) - \frac{1}{2}\beta_0\phi^2mn^2/\kappa_{11} - \frac{1}{4}\beta m\,{}_0\phi^2$$

$$+\frac{1}{2}Umn^2\,{}_0\phi^2+\frac{1}{2}c_0\mu^{-2}m^3/\kappa_{11}\cdot{}_0\phi^2\Big](s_2c_0)$$
$$+\left[\frac{1}{4}\beta m\,{}_0\phi^2+\frac{1}{2}c_0\mu^{-2}m\,{}_0\phi^2\right](s_2c_2)$$
$$+\left[-\frac{1}{4}m^3n\,{}_0\phi^3\right](c_1c_3)+\frac{1}{4}mn^3\cdot{}_0\phi^3(c_3c_1)$$
$$-2c_0m\cdot{}_1d_1(s_2c_0)e^{l\eta}-2c_0m\cdot{}_1d_2(s_2c_0)e^{-l\eta}\Big\} \tag{23}$$

由此得到(19)式的一个特解

$$\begin{aligned}\tilde{\chi}_2=\ &\alpha_1(c_1s_1)+\alpha_2(c_1c_1)+\alpha_3(c_1s_1)\eta+\alpha_4(c_1c_1)\eta+\alpha_5(c_1c_1)\eta^2\\&+\alpha_6(s_2c_0)+\alpha_7(s_2c_2)+\alpha_8(c_1c_3)+\alpha_9(c_3c_1)\\&+\alpha_{10}(s_2c_0)e^{l\eta}+\alpha_{11}(s_2c_0)e^{-l\eta}\end{aligned} \tag{24}$$

其中 α_1—α_{11} 的表达式见附录 1。

于是可将(19)—(20)式转化为 Laplace 方程边值问题

$$\begin{cases}\Delta\chi_2'=0 & (19)'\\ \left.\dfrac{\partial\chi_2'}{\partial\eta}\right|_{\eta=0,\pi}=-\left.\dfrac{\partial\tilde{\chi}_2}{\partial\eta}\right|_{\eta=0,\pi} & (20)'\end{cases}$$

可求得

$$\chi_2'=\alpha_{12}(c_1c_0)e^{m\eta}+\alpha_{13}(c_1c_0)e^{-m\eta}+\alpha_{14}(s_2c_0)e^{2m\eta}+\alpha_{15}(s_2c_0)e^{-2m\eta} \tag{25}$$

其中

$$\begin{cases}\alpha_{12}=[(-1)^{n+1}(n\alpha_1+n\pi\alpha_3+\alpha_4+2\pi\alpha_5)+e^{-m\pi}(n\alpha_1+\alpha_4)]/m(e^{m\pi}-e^{-m\pi})\\ \alpha_{13}=[(-1)^{n+1}(n\alpha_1+n\pi\alpha_3+\alpha_4+2\pi\alpha_5)+e^{m\pi}(n\alpha_1+\alpha_4)]/m(e^{m\pi}-e^{-m\pi})\\ \alpha_{14}=\dfrac{1}{2}l[(e^{-l\pi}-e^{-2m\pi})\alpha_{11}-(e^{l\pi}-e^{-2m\pi})\alpha_{10}]/m(e^{2m\pi}-e^{-2m\pi})\\ \alpha_{15}=l[(e^{-l\pi}-e^{2m\pi})\alpha_{11}-(e^{l\pi}-e^{2m\pi})\alpha_{10}]/m(e^{2m\pi}-e^{-2m\pi})\end{cases} \tag{26}$$

最后得到速度势的二级修正

$$\chi_2=\tilde{\chi}_2+\chi_2' \tag{27}$$

为了给出散度场的一级和二级修正，我们取定一组原始特征量：$f_0=10^{-4}$ 秒$^{-1}$，$\left(\dfrac{df}{dy}\right)=0.56\times10^{-11}$ 米$^{-1}$秒$^{-1}$，$\tilde{\phi}=5000\times9.81$ 米^{2}秒$^{-2}$，$L^*=10^6$ 米。对于准地转

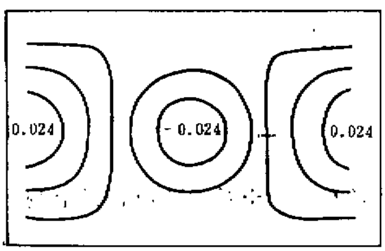

图 1　散度场的一级近似 $\varepsilon\Delta\chi_1 t^{*-1}$
(单位：10^{-5}秒$^{-1}$)

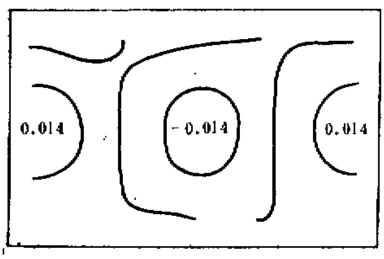

图 2　散度场的二级修正 $\varepsilon^2\Delta\chi_2 t^{*-1}$
(单位：10^{-5}秒$^{-1}$)

性参数 $\varepsilon = 0.3$ 的情形，推得：$t^* = \frac{1}{3} \times 10^5$ 秒，$U^* = 30$ 米秒$^{-1}$，$\Phi^* = 3000$ 米^{2}秒$^{-2}$。再取定零级和一级近似中的自由参数 $U = 0.5$，${}_0\psi = 0.35$，$m = n = 1$，${}_1U_1 = 0.5$，就可以由(7)式右端及(23)式进行数值计算。

图 1 和图 2 分别是已恢复量纲的 $\varepsilon\Delta\chi_1$ 和 $\varepsilon^2\Delta\chi_2$（以后给出的图均已如此处理），它们和[2]中给出的结果是类似的，但强度都明显加大了。散度的二级修正可达一级近似的 60%，是散度场中不可忽略的一部分。

三、散度方程的求解

将(21)式右端写成

$$Q_2 = \Delta\tilde{A}_2^{(1)} + Q_2^{(2)} \tag{28}$$

其中

$$\tilde{A}_2^{(1)} = c_0\frac{\partial\chi_1}{\partial\xi} + \beta\eta\psi_1 - \nabla\psi_0\cdot\nabla\psi_1 - J(\psi_0,\ \chi_1) \tag{29}$$

$$\begin{aligned} Q_2^{(2)} = &-\beta\frac{\partial\chi_1}{\partial\xi} - \beta\frac{\partial\psi_1}{\partial\eta} + 2\Delta\psi_0\Delta\psi_1 + \nabla\psi_1\cdot\nabla(\Delta\psi_0) \\ &+ \nabla\psi_0\cdot\nabla(\Delta\psi_1) - J(\chi_1,\ \Delta\psi_0) \end{aligned} \tag{30}$$

分别计算 $\tilde{A}_2^{(1)}$ 和 $Q_2^{(2)}$，再由 $Q_2^{(2)}$ 对 Laplace 算子求逆，可得到(21)式的一个特解

$$\begin{aligned} \tilde{A}_2 = &\beta_0 + \beta_1\eta + \beta_2\eta^2 + \beta_3\eta^3 + \beta_4(s_1s_1) + \beta_5(s_1c_1) + \beta_6(s_1s_1)\eta \\ &+ \beta_7(s_1c_1)\eta + \beta_8(s_1s_1)\eta^2 + \beta_9(s_1c_1)\eta^2 + \beta_{10}(s_1c_1)\eta^3 \\ &+ \beta_{11}(c_2c_0) + \beta_{14}(c_0s_2) + \beta_{15}(c_2s_2) + \beta_{16}(c_2c_0)\eta \\ &+ \beta_{17}(c_0c_2)\eta + \beta_{19}(c_0s_2)\eta + \beta_{21}(c_0s_2)\eta^2 + \beta_{23}(s_3s_1) \\ &+ \beta_{24}(s_1s_3) + \beta_{25}(c_2c_0)e^{l\eta} + \beta_{26}(c_2c_0)e^{-l\eta} + \beta_{27}(c_2c_0)\eta e^{l\eta} \\ &+ \beta_{28}(c_2c_0)\eta e^{-l\eta} + \beta_{29}(s_1s_1)e^{l\eta} + \beta_{30}(s_1s_1)e^{-l\eta} \\ &+ \beta_{31}(s_1c_1)e^{l\eta} + \beta_{32}(s_1c_1)e^{-l\eta} + \beta_{33}(s_3s_1)e^{l\eta} + \beta_{34}(s_3s_1)e^{-l\eta} \\ &+ \beta_{35}(s_3c_1)e^{l\eta} + \beta_{36}(s_3c_1)e^{-l\eta} + \beta_{37}(s_1c_0)e^{m\eta} + \beta_{38}(s_1c_0)e^{-m\eta} \end{aligned} \tag{31}$$

其中 β_0—β_{38} 的表达式见附录 2。

这样可将问题(21)—(22)转化为

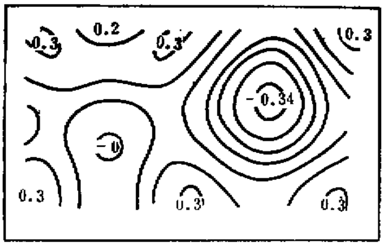

图 3　非地转涡度的一级近似 $\varepsilon\Delta A_1 t^{*-1}$
（单位：10^{-5}秒$^{-1}$）

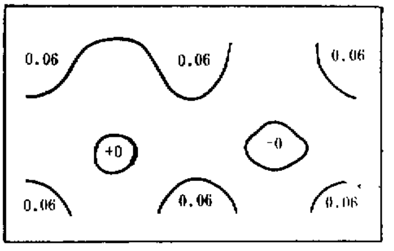

图 4　非地转涡度的二级修正 $\varepsilon^2\Delta A_2 t^{*-1}$
（单位：10^{-7}秒$^{-1}$）

$$\begin{cases} \Delta A_2' = 0 & (21)' \\ \left.\dfrac{\partial A_2'}{\partial \eta}\right|_{\eta=0,\pi} = -\left[\dfrac{\partial \tilde{A}_2}{\partial \eta} + \dfrac{\partial \chi_2}{\partial \xi} + \beta\eta\left(\dfrac{\partial \chi_1}{\partial \xi} - \dfrac{\partial \psi_1}{\partial \eta}\right)\right]_{\eta=0,\pi} & (22)' \end{cases}$$

可解得

$$\begin{aligned} A_2' = {} & \beta_1'\eta + \beta_{37}'(s_1c_0)e^{m\eta} + \beta_{38}'(s_1c_0)e^{-m\eta} + \beta_{39}'(c_2c_0)e^{2m\eta} + \beta_{40}(c_2c_0)e^{-2m\eta} \\ & + \beta_{41}(s_3c_0)e^{3m\eta} + \beta_{42}(s_3c_0)e^{-3m\eta} \end{aligned} \tag{32}$$

其中

$$\begin{cases} \beta_1' = -(\beta_1 + 2n\beta_{14} + \beta_{17}) \\ \beta_{37}' = (b_{\pi 1} - e^{-m\pi}b_{01})/m(e^{m\pi} - e^{-m\pi}) \\ \beta_{38}' = (b_{\pi 1} - e^{m\pi}b_{01})/m(e^{m\pi} - e^{-m\pi}) \\ \beta_{39} = (b_{\pi 2} - e^{-2m\pi}b_{02})/2m(e^{2m\pi} - e^{-2m\pi}) \\ \beta_{40} = (b_{\pi 2} - e^{2m\pi}b_{02})/2m(e^{2m\pi} - e^{-2m\pi}) \\ \beta_{41} = (b_{\pi 3} - e^{-3m\pi}b_{03})/3m(e^{3m\pi} - e^{-3m\pi}) \\ \beta_{42} = (b_{\pi 3} - e^{3m\pi}b_{03})/3m(e^{3m\pi} - e^{-3m\pi}) \end{cases} \tag{33}$$

而

$$\begin{cases} b_{00} = -(\beta_1 + 2n\beta_{14} + \beta_{17}) \\ b_{01} = m(\alpha_2 + \alpha_8 + \alpha_{12} + \alpha_{13}) - (n\beta_4 + \beta_7 + 3n\beta_{14} + n\beta_{29} \\ \qquad + n\beta_{30} + l\beta_{31} - l\beta_{32} + m\beta_{37} - m\beta_{38}) \\ b_{02} = -2m(\alpha_6 + \alpha_7 + \alpha_{10} + \alpha_{11} + \alpha_{14} + \alpha_{15}) \\ \qquad - (2n\beta_{15} + \beta_{16} + l\beta_{25} - l\beta_{26} + \beta_{27} + \beta_{28}) \\ b_{03} = 3m\alpha_9 - (n\beta_{23} + n\beta_{33} + n\beta_{34} + l\beta_{35} - l\beta_{36}) \\ b_{\pi 0} = -\pi\beta({}_1U_1 + 2\pi \cdot {}_1U_2) - (\beta_1 + 2\pi\beta_2 + 3\pi^2\beta_3 + 2n\beta_{14} + \beta_{17} \\ \qquad + 2n\pi\beta_{19} + 2\pi^2 n\beta_{21}) \\ b_{\pi 1} = (-1)^n[m(\alpha_2 + \pi\alpha_4 + \pi^2\alpha_5 + \alpha_8) + {}_1B_0 + \pi(n \cdot {}_1B_1 + 2 \cdot {}_1B_2)] \\ \qquad + m(e^{m\pi}\alpha_{12} + e^{-m\pi}\alpha_{13}) - (-1)^n[n\beta_{14} + \pi n\beta_6 + \beta_7 \\ \qquad + \pi^2 n\beta_8 + 2\pi\beta_9 + 3\pi^2\beta_{10} + 3n\beta_{24} + ne^{l\pi}\beta_{29} + ne^{-l\pi}\beta_{30} \\ \qquad + le^{l\pi}\beta_{31} - le^{-l\pi}\beta_{32} + (-1)^n me^{m\pi}\beta_{37} - (-1)^n me^{m\pi}\beta_{38}] \end{cases} \tag{34}$$

最后得到非地转位势的二级修正

$$A_2 = \tilde{A}_2 + A_2' \tag{35}$$

图 3 和图 4 是由前述参数值算得的非地转涡度的一级近似和二级修正，这两者的分布仅在区域的边界附近类似，而后者比前者小得多(在区域内部更小)。

以上，我们为求得 χ_2 和 A_2 不得不进行大量的演算，这种情况在求解 ψ_2 时更加严重。在本文的第(二)部分中，我们将结合位涡度方程的求解给出一种克服这个困难的办法。

致谢：本文是在曾庆存同志的鼓励与指导下完成的。吴津生同志帮助作者发现了求解散度方程演算中的一个重要错误，作者深为感谢。

附录 1　α_1—α_{11} 的表达式

$$\alpha_1 = \mu^{-2}m/\kappa_{11}\cdot[c_0\cdot {}_0\psi + 2c_0 n\cdot {}_1B_0/\kappa_{11}]$$

$$\alpha_2 = \mu^{-2}\kappa_{11}[c_0 m({}_1X_1^+ - {}_1X_1^- - \beta n/\kappa_{11}\cdot {}_0\psi) + \frac{1}{4}mn(m^2-n^2)\cdot {}_0\psi^2 - U(c_0\mu^{-2}-\beta)mn/\kappa_{11}\cdot {}_0\psi$$
$$- 2c_0 mn/\kappa_{11}(\beta\cdot {}_0\psi + {}_1B_1) + 2c_0 m(m^2-3n^2)/\kappa_{11}^2\cdot {}_1B_2]$$

$$\alpha_3 = \mu^{-2}/\kappa_{11}[c_0 m(\beta\cdot {}_0\psi + {}_1B_1) + Uc_0\mu^{-2}m\cdot {}_0\psi + 4c_0 mn/\kappa_{11}\cdot {}_1B_2]$$

$$\alpha_4 = \mu^{-2}/\kappa_{11}\cdot c_0 m\cdot {}_1B_0$$

$$\alpha_5 = \mu^{-2}/\kappa_{11}\cdot c_0 m\cdot {}_1B_2$$

$$\alpha_6 = \mu^{-2}/m\left[\frac{1}{2}c_0\left({}_1\psi_{20}^{cc} + \frac{1}{4}n^2\cdot {}_0\psi^2\right) + \frac{1}{8}\beta n^2/\kappa_{11}\cdot {}_0\psi^2 + \frac{1}{16}\beta\cdot {}_0\psi^2 - \frac{1}{8}Un^2\cdot {}_0\psi^2\right.$$
$$\left. - \frac{1}{8}c_0\mu^{-2}m^2/\kappa_{11}\cdot {}_0\psi^2\right]$$

$$\alpha_7 = \mu^{-2}/\kappa_{11}\left[\frac{1}{16}\beta m/\kappa_{11}\cdot {}_0\psi^2 + \frac{1}{8}c_0\mu^{-2}m/\kappa_{11}\cdot {}_0\psi^2\right]$$

$$\alpha_8 = \mu^{-2}/(m^2+9n^2)\cdot\left[\frac{1}{4}m^3 n\cdot {}_0\psi^3\right]$$

$$\alpha_9 = \mu^{-2}/(9m^2+n^2)\cdot\left[-\frac{1}{4}mn^3\cdot {}_0\psi^3\right]$$

$$\alpha_{10} = \mu^{-2}/\kappa_{11}[-2c_0 m\cdot {}_1d_1]$$

$$\alpha_{11} = \mu^{-2}/\kappa_{11}[-2c_0 m\cdot {}_1d_2]$$

附录 2　β_0—β_{34} 的表达式

$$\beta_0 = -U\cdot {}_1U_1 - \frac{1}{4}n\cdot {}_1B_0\cdot {}_0\psi$$

$$\beta_1 = \frac{1}{4}\kappa_{11}\cdot {}_1B_1\cdot {}_0\psi - 2U\cdot {}_1U_2 - \frac{1}{2}n\cdot {}_1B_2\cdot {}_0\psi$$

$$\beta_2 = -\frac{1}{2}\beta\cdot {}_1U_1$$

$$\beta_3 = -\frac{2}{3}\beta\cdot {}_1U_2$$

$$\beta_4 = 1/\kappa_{11}\cdot[c_0\mu^{-4}m^2/\kappa_{11}\cdot {}_0\psi + 4m^2\cdot {}_1U_2\cdot {}_0\psi + \beta n({}_1X_1^+ - {}_1X_1^-) + \beta(m^2-n^2)/\kappa_{11}\cdot {}_1B_1$$
$$+ 2\beta n(3m^2-n^2)/\kappa_{11}^2\cdot {}_1B_2 - 4({}_1\Psi_{20}^{cc} + {}_1\Psi_{02}^{cc})m^2n^2\cdot {}_0\psi]$$

$$\beta_5 = 1/\kappa_{11}^2\cdot[\beta\kappa_{11}\cdot {}_1B_0 - 2\beta n^2\cdot {}_1B_0]$$

$$\beta_6 = 1/\kappa_{11}\cdot\beta n\cdot {}_1B_0$$

$$\beta_7 = \beta({}_1X_1^+ - {}_1X_1^-) - \beta n/\kappa_{11}\cdot {}_1B_1 - 2\beta(n^2-m^2)/\kappa_{11}^2\cdot {}_1B_2$$

$$\beta_8 = \beta\cdot {}_1B_1 + \beta n/\kappa_{11}\cdot {}_1B_2$$

$$\beta_9 = \beta\cdot {}_1B_0$$

$$\beta_{10} = \beta\cdot {}_1B_2$$

$$\beta_{11} = \frac{1}{2}\beta n\cdot {}_0\psi$$

$$\beta_{14} = \frac{1}{8}c_0\mu^{-2}m^2(m^2-3n^2)/n/\kappa_{11}\cdot {}_0\psi - \frac{1}{2}({}_1X_1^+ - {}_1X_1^-)m^2{}_0\psi - \frac{1}{2}\beta/n\cdot {}_1\Psi_{02}^{cc}$$

$$\beta_{15} = \frac{1}{4}m^2/\kappa_{11}\cdot {}_1B_2\cdot {}_0\psi$$

$$\beta_{16} = \frac{1}{2}n^2\cdot {}_1B_1\cdot {}_0\psi + n\cdot {}_1B_2\cdot {}_0\psi + \beta\cdot {}_1\Psi_{20}^{cc}$$

$$\beta_{17} = \frac{1}{2}m^2\cdot {}_1B_1\cdot {}_0\psi + \beta\cdot {}_1\Psi_{02}^{cc}$$

$$\beta_{19} = -\frac{1}{2} m^2 \cdot {}_1B_0 \cdot {}_0\psi$$

$$\beta_{21} = -\frac{1}{2} m^2 \cdot {}_1B_2 \cdot {}_0\psi$$

$$\beta_{23} = -4m^2n^2/(9m^2 + n^2) \cdot {}_1\Psi^{cc}_{20} \cdot {}_0\psi$$

$$\beta_{24} = -4m^2n^2/(m^2 + 9n^2) \cdot {}_1\Psi^{cc}_{02} \cdot {}_0\psi$$

$$\beta_{25} = -\beta l/\kappa_{11}$$

$$\beta_{26} = \beta l/\kappa_{11}$$

$$\beta_{27} = \beta \cdot {}_1d_1$$

$$\beta_{28} = \beta \cdot {}_1d_2$$

$$\beta_{29} = \frac{1}{2}(3m^2 + n^2) \cdot {}_1d_1 \cdot {}_0\psi$$

$$\beta_{30} = \frac{1}{2}(3m^2 + n^2) \cdot {}_1d_2 \cdot {}_0\psi$$

$$\beta_{31} = -\frac{1}{2} nl \cdot {}_1d_1 \cdot {}_0\psi$$

$$\beta_{32} = -\frac{1}{2} nl \cdot {}_1d_2 \cdot {}_0\psi$$

$$\beta_{33} = \frac{1}{2}(m^2 - n^2) \cdot {}_1d_1 \cdot {}_0\psi$$

$$\beta_{34} = \frac{1}{2}(m^2 - n^2) \cdot {}_1d_2 \cdot {}_0\psi$$

$$\beta_{35} = -\frac{1}{2} nl \cdot {}_1d_1 \cdot {}_0\psi$$

$$\beta_{36} = \frac{1}{2} nl \cdot {}_1d_2 \cdot {}_0\psi$$

$$\beta_{37} = -c_0 m \cdot {}_1X_1^+$$

$$\beta_{38} = -c_0 m \cdot {}_1X_1^-$$

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SECOND APPROXIMATIONS TO NONLINEAR WAVES IN THE BAROTROPIC ATMOSPHERE (I)

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Abstract

This is the first part of the work on seeking the second order approximations to the large scale wave solutions of the primitive equations by perturbation method. In this part, the second order revisions for the velocity potential and the ageostrophic geopotential are obtained by solving the vorticity and divergence equations.